Flow Starting Times in Constant-Area Supersonic Diffusers

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The simplest supersonic diffusers are constant-area ducts. This paper is concerned mainly with the time period required to establish steady flow from rest in a supersonic nozzle followed by a straight-duct, constant-area 5.1×5.1 cm^2 diffuser. The experiments are carried out in a Ludwieg tube with a diaphragm location either upstream or downstream of the test section. The flow conditions at the nozzle exit are: Mach number $M = 2.8$ and Reynolds number $Re_D = 3.6 \times 10^5$, as based on hydraulic diameter and freestream conditions. The starting times for the flow at the nozzle exit are greatly affected by the position of the diaphragm. For both diaphragm locations, the flow starting times at the nozzle exit increase with the diffuser length ($0 \leq L/D \leq 6$). The optimum pressure ratio p_d/p_0 for the steady flow to start at the nozzle exit is obtained for a diffuser length of $L/D \sim 5.5$. This length also corresponds to the conditions for optimum pressure recovery during the steady flow cycle of the Ludwieg tube.

Nomenclature

A = area, nozzle exit area, 5.1×5.1 cm^2
D = hydraulic diameter at nozzle exit (four times cross-sectional area divided by perimeter), 5.1 cm
L = diffuser length
M = Mach number
p = pressure
Re_D = Reynolds number based on hydraulic diameter and test-section freestream parameters, $(v\rho D)/\mu$
t_s = starting time for the flow at a given location, defined as the time period between the arrival of the head of the expansion fan (downstream diaphragm), or shock wave (upstream diaphragm) until steady flow is established
v = flow speed
x = axial distance, $x = 0$ at nozzle exit
δ^* = boundary-layer displacement thickness
τ_f = characteristic flow time

Subscripts

1 = downstream initial conditions before diaphragm rupture
4 = upstream initial conditions before diaphragm rupture
3 = conditions upstream of nozzle after the expansion fan has passed into the supply tube
0 = reservoir, nozzle supply conditions during steady flow cycle of the Ludwieg tube
d = diffuser exit conditions during steady flow cycle in the Ludwieg tube

I. Introduction

THE pressure recovery of supersonic diffusers following narrow ducts, as encountered in novel applications such as gasdynamic lasers, cannot be improved markedly by refined diffuser designs.[1] This is especially true if the diffuser does not operate at the proper design specifications. Parallel rows of such diffusers have been proposed for gasdynamic lasers,[2] thus the simplicity of the diffuser becomes a key factor in practical applications. Hence the straight-duct supersonic diffuser emerges as an attractive possibility, and new interest in its performance under various conditions has arisen.

The most obvious performance criterion of a diffuser is the optimum pressure recovery that it achieves operating at its design point. Moreover, the required diffuser length to obtain optimum pressure recovery is of importance. These two topics have been investigated in earlier years by Neumann and Lustwerk[3] and more recently by Waltrup and Billig[4] and Davidson and Chukhalo.[5] The work was extended recently by Merkli[6,7] for narrow channels. An additional important performance criterion is the time required to get the flow started, that is, the time required to achieve a steady flow from rest in the supersonic nozzle. This is the main topic of the present paper.

The experiments were carried out in a Ludwieg tube,[8,9] which consists of a shock tube with a converging-diverging nozzle to generate supersonic flow in the test section. It is a simple intermittent supersonic wind tunnel. The diaphragm separating the driver and driven sections can be located either downstream of the diffuser or upstream of the nozzle.[10] For downstream diaphragm locations, the flow starting time in the nozzle is known to depend upon the nozzle geometry and the diaphragm location.[11-13] The starting time of supersonic flow at a given location is defined as the period between the arrival of the head of the expansion fan (downstream diaphragm), or shock wave (upstream diaphragm), and the instance where the pressure becomes time-independent, i.e., steady flow is established. For an upstream diaphragm location, the starting times at the nozzle exit are reported to be shorter than those obtained with the downstream diaphragm location.[10,14]

The present paper reports experiments on starting times for identical geometrical nozzle-diffuser arrangements with diaphragm locations both upstream and downstream of the Ludwieg tube test section. The starting time was measured as a function of the diffuser length L and the downstream pressure. Results on pressure recovery are compared with those of previous experiments conducted in a smaller continuous wind tunnel with a supersonic nozzle of similar design.[6,7]

II. Experimental

The Ludwieg tube used in our experiments has the following dimensions: length of driver section 670 cm, diameter of driver section 13.4 cm, length of driven section 760 cm, and diameter of driven section 9.5 cm. With the speed of sound roughly ⅓ m/msec, each meter of tube length of the

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driver section gives approximately 6 msec of steady running time in a given cycle. The driven section, terminating in a dump tank of volume 4.8×10^5 cm^3, exhibits even longer cycles. Thus the running time of the first flow cycle in our facility (roughly 40 msec) is much longer than the observed starting times for the nozzle/diffuser flow.

In contrast to the tubes, the test section has rectangular cross section 5.1×12.7 cm^2, thus necessitating a transition piece on either side. The overall length of the test section is 61 cm, of which 30.5 cm were used by the nozzle, leaving another 30.5 cm for various supersonic diffusers. The nozzle has a throat of 1.2×5.1 cm^2 located 10.2 cm from the nozzle inlet and a square exit area of 5.1×5.1 cm^2. The supersonic nozzle contour was based on calculations using the method of characteristics for a uniform $M=3$ flow. No boundary-layer corrections are applied, since no fixed design Reynolds number was used. Pressure measurements are made with Kistler piezo quartz transducers, model 606L, mounted in the test section side wall, e.g., along the centerline with membranes flush to the channel wall. The diaphragm can be located at either end of the test section. Two layers of 0.005-cm-thick cellophane sheets were used as diaphragms in the experiments conducted with the downstream diaphragm location. The diaphragm in this case was ruptured by a spring-actuated pin mechanism. In the experiments conducted with an upstream diaphragm location, a 0.007-cm-thick mylar sheet was used. The mylar sheets, which were scratched along diagonal lines with a razor blade, were ruptured by an X-shaped cutter.‡ The Ludwieg tube was triggered manually, whereas the four-channel storage oscilloscope, Tektronix model 5103N, was triggered by breaking a circuit when releasing the pin to rupture the diaphragm, or, with upstream diaphragm locations, by the signal of a pressure transducer located at the nozzle entrance.

Bottled dry air (dew point of 213 K maximum) was used as test gas. All runs were made with an initial upstream pressure $p_4 = 700$ Torr, resulting in a supply pressure $p_3 = 675$ Torr during the steady flow cycle.§ The initial temperature in the tube, $T_4 = 296$ K, leads to a supply temperature of $T_3 = 293$ K during the steady flow cycle. The subscript 3 refers to the static parameters after the expansion fan has passed into the supply tube. Since, in our Ludwieg tube, the flow speed in the supply tube after the passage of the expansion fan is very low, i.e., $M_3 = 0.02$, $p_3 \cong p_0$ and $T_3 \cong T_0$, where p_0 and T_0 are the supply pressure and temperature of the nozzle during the steady flow cycle.

Figures 1a and 1b depict typical pressure traces as functions of time taken in the middle of the wall at the nozzle exit ($x=0$), with a 15.2-cm diffuser following. For a downstream diaphragm location (Fig. 1a), the pressure abruptly decreases from its initial value with the arrival of the expansion fan and levels off at the new steady value. The small pressure bump observed in these pressure traces is known to be caused by the starting shock.[11] The pressure trace of Fig. 1b is recorded with an upstream diaphragm location. The pressure increases from its initial low value after the arrival of the shock wave and then decreases and levels off at the steady value. The starting times t_s as defined previously are marked in Figs. 1a and 1b.

From static pressure measurements, the nozzle exit Mach number was computed to be $M=2.8$ (± 0.2). Under the current conditions ($T_0 = 293$ K, $p_0 = 675$ Torr), the Reynolds number at the nozzle exit is $Re_D = 3.6 \times 10^5$, and the boundary-layer displacement parameter is of the order of $\delta^*/D \sim 0.04$. Thus the flow parameters are within the range of previous investigations carried out at this laboratory with a continuous tunnel.[6,7]

‡This prevents diaphragm pieces from flying through the test section.

§For expressions relating these properties, note references on the Ludwieg tube.[9]

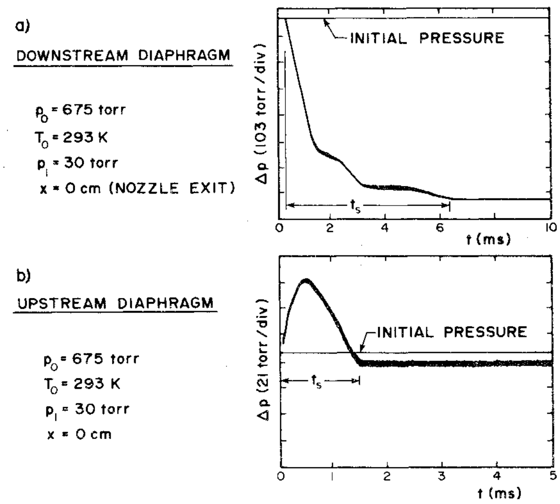

Fig. 1 Typical traces of static pressure at the nozzle exit as a function of time (diffuser length, $L/D=3$). Note definition of starting time.

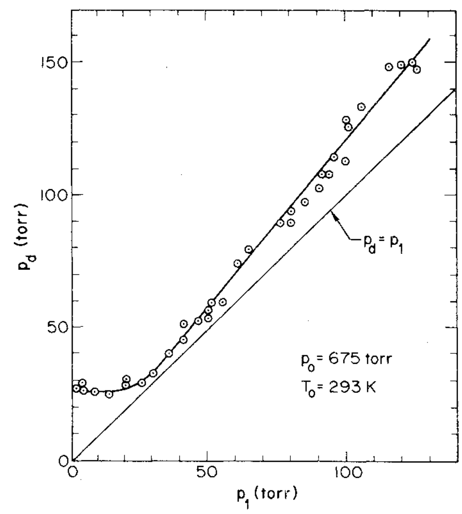

Fig. 2 Diffuser exit pressure p_d as a function of the initial downstream pressure p_I ($L/D=3$; downstream diaphragm location).

III. Results

A. Downstream Diaphragm Location

Diffuser Exit Pressure During the Steady Flow Cycle

After breaking the diaphragm, first a complicated unsteady flow develops in the test section until the first steady flow cycle is reached. The pressure downstream of the diffuser for a started flow, p_d, is then different from the initial pressure in the driven Ludwieg tube section p_I. Thus, to determine the pressure recovery of the diffuser flow, the relation between the two pressures, p_I and p_d, has to be known. Such measurements were performed mainly for a diffuser of length 15.2 cm. As seen from Fig. 2, the diffuser exit pressure p_d is higher than the initial pressure p_I and increases linearly with the initial downstream pressure, except at low values of p_I, where p_d leveled off. Since this behavior is practically independent of the diffuser lengths investigated here, the results of Fig. 2 can be applied to all of our experiments.

Starting Times

In Fig. 3, starting times at the nozzle and diffuser ends are given for a 15.2-cm-long straight-duct diffuser (see also Ref. 15). It is seen that the starting times are independent of the initial downstream pressure p_I except in a narrow range at the performance limit. Thus they can be plotted as a function of

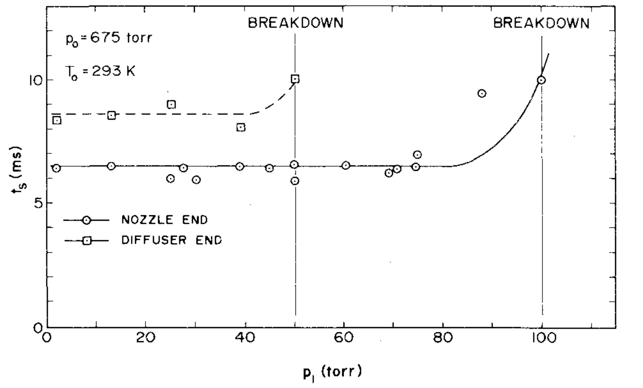

Fig. 3 Starting time for the flow at the nozzle and diffuser exits as a function of initial downstream pressure p_l ($L/D=3$; downstream diaphragm location).

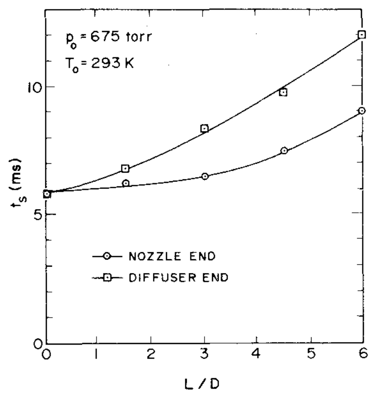

Fig. 4 Starting times for supersonic flow at the nozzle and diffuser exits as a function of L/D. The downstream pressure is low enough to allow supersonic flow in the nozzle-diffuser section (downstream diaphragm location).

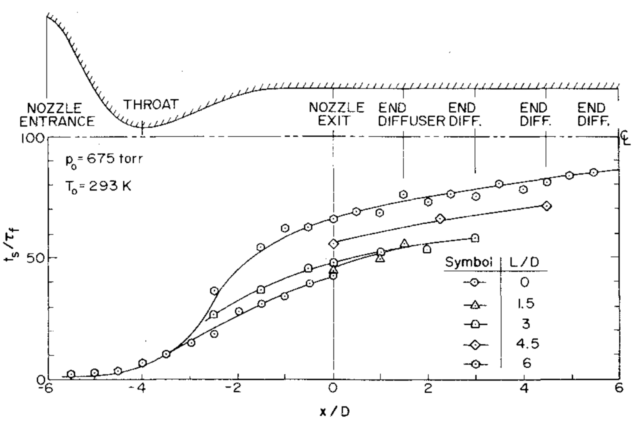

Fig. 5 Dimensionless starting time (t_s/τ_f) as a function of dimensionless axial distance from the exit of the nozzle (x/D) for various diffuser lengths (downstream diaphragm location). The downstream pressure is low enough to allow supersonic flow in the nozzle-diffuser section.

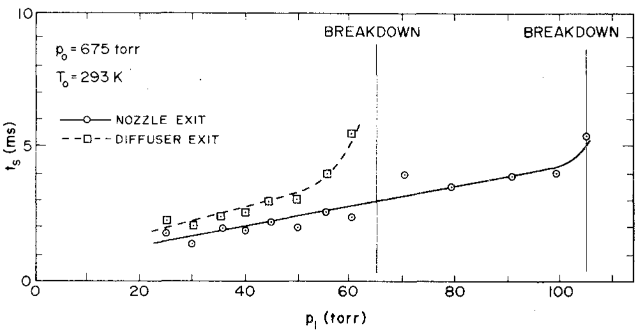

Fig. 6 Starting times for the supersonic flow at the nozzle and diffuser exits as a function of the initial downstream pressure p_l ($L/D=3$; upstream diaphragm location). Nozzle entrance at 4.6 cm from diaphragm.

the diffuser length alone, as shown in Fig. 4. We learn from this diagram that the required flow starting time increases with diffuser length both at the nozzle and diffuser exit. In dimensionless form, Fig. 5 also shows the starting times within the nozzle and the diffuser. The starting times upstream of the nozzle throat do not depend noticeably upon the diffuser length. In the diverging section of the nozzle, the starting times for the flow increase 1) with distance from the throat, and 2) with following length of the diffuser. The time scale $\tau_f = \ell/a^*$ and length $\ell = (R^*h^*)^{1/2}$ as used in Fig. 5 are related directly to the flow speed increase and temperature gradient in the nozzle throat.[16] R^* is the radius of curvature, h^* the nozzle height, and a^* the speed of sound, all taken at the nozzle throat. In our experiment $R^* = 19.4$ cm, $h^* = 1.2$ cm, and $\tau_f = 0.15$ msec.

B. Upstream Diaphragm Location

Also in this configuration, the flow starting times were recorded as a function of the initial downstream pressure p_l and diffuser length L. Since the rupture of the mylar diaphragm is not reproducible, resulting in small variations of opening times of a few tenths of a millisecond, a certain scatter in the experimental data is noted. However, this is unimportant, since the flow starting times are of the order of a few milliseconds.

Figure 6 shows starting times as measured at the nozzle and diffuser exits as a function of the initial downstream pressure p_l. Here the nozzle was followed by a diffuser of length 15.2 cm ($L/D=3$). Similar results were obtained for other straight-duct diffusers with $L/D=0$, 1.5, 4.5, and 6.[15] As already reported by Falk and Hertzberg[10] and Davis,[14] a comparison with previous results shows that starting times are indeed shorter with an upstream location. Then, in contrast to the experiments with downstream diaphragm location, the flow starting times here are observed to increase with the initial downstream pressure p_l. This trend prevails throughout the range of downstream pressures, allowing the supersonic flow to start. Hence the starting times for this configuration cannot be summarized as concisely as a function of L/D only, as in the earlier case. Results for other diffuser lengths are shown in Fig. 7. It is seen here that, for a fixed value of the initial downstream pressure p_l, the flow starting time increases with the diffuser length.

C. Starting Pressure Ratios and Pressure Recovery during the Steady Ludwieg Tube Flow Cycle

For a fixed geometry and supply pressure, the supersonic flow will be established throughout the entire nozzle-diffuser assembly if the downstream pressure is low enough. With the back pressure being increased stepwise at a fixed supply pressure, the flow at the diffuser end ceases to start for a certain value of p_d. However, the flow at the nozzle end still is unaffected. A further increase of the downstream pressure leads to a second operating limit. This limit is attained if the downstream pressure is so high that the supersonic flow at the nozzle exit barely starts. Beyond this critical condition, the flow at the nozzle exit becomes unstable, and a succession of starts and flow breakdowns is noted. At this limit, we have imposed the maximum pressure recovery that the diffuser can sustain.

In Fig. 8, these limiting downstream pressure ratios p_d/p_0 are plotted as a function of the diffuser length L/D for both upstream and downstream diaphragm locations. The optimum pressure ratio p_d/p_0 for the flow to still start at the nozzle exit is achieved by a diffuser with $L/D \sim 5.5$. For this

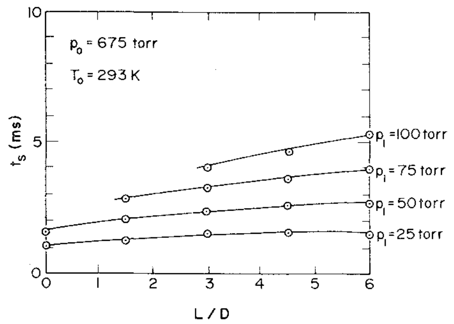

Fig. 7 Starting times for supersonic flow at the nozzle exit as a function of L/D for various initial downstream pressures p_i (upstream diaphragm location). Nozzle entrance at 4.6 cm from diaphragm.

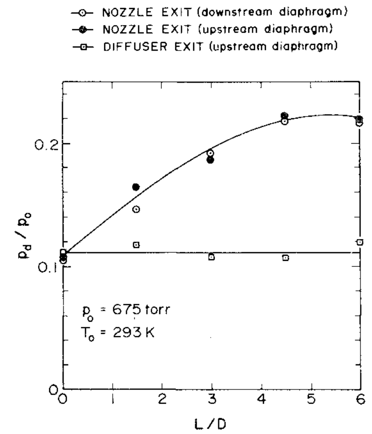

Fig. 8 Limiting pressure ratios p_d/p_0 for the supersonic flow to start properly at the nozzle and diffuser exits as a function of L/D.

geometry, the diffuser also operates with the maximum pressure recovery in steady flow, since, under the prevailing conditions, the diffuser contains the full pressure recovery zone but no further duct section after the recovery zone which could add frictional losses. This optimum value of L/D is in agreement with previous work by Merkli,[6,7] where for similar conditions a diffuser of $L/D \sim 6$ was found to yield maximum pressure recovery for continuous wind-tunnel operation. In the same work, it was found moreover that an established flow breaks down at the nozzle exit if the back pressure is increased to a value of $p_d/p_0 = 0.24$. Here it is seen that, for a diffuser length of $L/D \sim 5.5$, the supersonic flow at the nozzle exit can be started properly for an initial pressure ratio $p_i/p_0 = 0.18$, corresponding to a pressure recovery of $p_d/p_0 = 0.22$, as follows using Fig. 2.¶ This indicates that the nozzle flow can still be started in the Ludwieg tube for a final pressure ratio quite close to the optimum one. In order to be sure of a clear flow throughout the nozzle for applications, it is recommended to operate at a somewhat lower pressure ratio p_d/p_0 than the limiting one presented here, so that small disturbances do not unstart the flow.

In Fig. 8 also, the less important limiting pressure ratios p_d/p_0, which cause the breakdown of the clean supersonic flow at the end of the test section, are shown for an upstream diaphragm location. In the case $L/D = 0$, the end of the test section is the nozzle end, or else it is the end of the straight diffuser following the nozzle. Within the accuracy of the experiments, this limit appears to be independent of the diffuser length $p_d/p_0 \sim 0.11$. Essentially, this means that for low enough back pressures even the longest diffuser with $L/D = 6$ has a negligible effect, i.e., on the velocity profile along x. Practically the same conditions prevail at the nozzle and diffuser end. Clearly not the same behavior can be expected for much longer flow channels. If the back pressure for the current arrangement is increased over the given limit, the necessary pressure recovery cannot be achieved outside of the flow channel anymore. Having no diffuser at all ($L/D = 0$), this means that the clean nozzle flow breaks down. With a diffuser, a certain recovery can then take place in this diffuser, as shown in Fig. 8, thus leaving the clean nozzle flow still unaffected.

¶Under the conditions when the flow almost breaks down at the nozzle exit, the pressure at the diffuser exit p_d shows violent fluctuations, thus rendering it difficult to determine the value of the pressure recovery with a high accuracy.

IV. Conclusions

1) The experiments on the diffuser flow starting times show that a) the flow starting times are shorter for the upstream diaphragm location than for the downstream diaphragm location; b) with the upstream diaphragm, the starting time increases with both downstream pressure and diffuser length; and c) for downstream diaphragm locations, the starting times are independent of the downstream pressure; however, they also increase with diffuser length.

2) The optimum pressure ratio p_d/p_0 to start the flow at the nozzle exit is obtained for a diffuser length of $L/D \sim 5.5$.

3) As in other facilities, the required pressure ratio p_d/p_0 to start the flow is lower than the one that breaks down an already established flow; yet the difference is small.

4) The diffuser performance during the steady flow cycle in a Ludwieg tube closely corresponds to results obtained in a steady flow facility using a similar nozzle-diffuser geometry.

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